IO-Snap

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IO-Snap Regionalization 2.0

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Abstract. Most input-output regionalization methods were developed before the wide-spread adoption of modern commodity-by-industry (CxI) input-output (IO) accounting frameworks. Correctly formulating the supporting accounting structures for national and regional analyses is essential, yet related textbook and journal articles often imply a simplicity that belies two important barriers to understanding. First, although modern IO data are now almost universally compiled and distributed in CxI format, presentations of IO methods are very commonly founded on inter*industry* accounts. Second, introductions to many IO-based methods tend to focus on national IO accounting, for which the accounting implications of open economic systems are seldom – if ever – discussed. This implies a simple and straightforward path from published national data to a coherent set of regional CxI accounts when, in fact, there are several key considerations to be taken and assumptions be made along the way. In this Technical Document, we lay out the mathematical foundations of a CxI version of traditional interindustry (IxI) regionalization and, in so doing, clarify appropriate regional commodity-by-industry impacts assessment formulations on which IO-Snap is founded. This document addresses reexports explicitly and supersedes 2020-1.

1 Preliminaries: Common Notation

IO-Snap follows the notational conventions of the commodity-by-industry input-output accounting framework shown in Figure 1, adapted from United Nations (1968).

Figure 1: The Commodity-Industry Framework

	Commodities	Industries	Final Demand	Totals
Commodities		U	e	q
Industries	V			g
Primary Inputs		va		
Totals	q'	g'		

- U = the *Use* table: row commodities used by column industries
- V = the *Make* table: column commodities produced by row industries
- e = column final demand activities use of row commodities
- q = column vector of total commodity use
- g = column vector of total industry output
- va = column industry payments to row value added components, typically compensation, payments to governments, and gross operating surplus
- $\mathbf{i}$ is a summing vector of appropriate dimension
- $'$ indicates transpose
- $\hat{}$ indicates diagonalization
- i, j => row and column sector subscripts. When used in combination, subscripts denote a from–to relationship, e.g., $z_{i,j}$ denotes a flow from source i to destination j
- r, s, N => superscripts denoting regions r and s, or national variables, N. When used in combination, superscripts denote origin and destination regions, e.g., z^{rs} denotes a flow from region r to region s

2 Regional Interindustry Accounts

As a point of reference, we begin with the general foundations for regionalizing interindustry IO accounts. Traditional regionalization methods in industry-by-industry space rely on the concept of regional supply percentages (RSP) that express a region's ability to satisfy its own demands. If we assume zero re-exports (exported imports) and let A be the matrix of interindustry direct requirements coefficients, where z_{ij} denotes industry j purchases from industry i and $a_{ij} \in A = \frac{z_{ij}}{X_j}$, regional output for *export* demand can be expressed as

$$(I - \hat{P}A)^{-1}E \tag{1}$$

and regional output for *regional* demand will be

$$(I - \hat{P}A)^{-1}\hat{P}(C + I + G), \tag{2}$$

where variables C, I, and G are regional consumption, investment, and government expenditures by industry, and regional supply percentages, P, are defined by

$$P = \widehat{(X - E + M)}^{-1}(X - E). \tag{3}$$

Variables E, M, and X are regional industry exports, imports, and output, respectively. The complete regional industry output balance equation, combining equations 1 and 2 can now be expressed as

$$X = (I - \hat{P}A)^{-1}(\hat{P}(C + I + G) + E), \tag{4}$$

which establishes the fundamental accounting relationships that form the basis of the standard interindustry impacts formulation,

$$\Delta X = (I - \hat{P}A)^{-1}[\hat{P}\Delta(C + I + G) + \Delta E] \tag{5}$$

Equation 5 might be useful for some in clarifying an area of common confusion in application, namely when and how to modify demands by RSP; they should modify all but export final demand. The confusion arises in part due to a tendency in many presentations to focus only on *components* of equation systems, e.g., coefficients matrices, multiplier matrices, and so on, without placing them in the context of the complete accounting systems equations. In the following section, we shift the focus to open regional economies and corresponding commodity by industry (CxI) accounts and regionalization methods.

3 Regionalizing CxI Frameworks

3.1 Closed Economy

Open and closed refer to trade status; closed economies are those that do not engage in trade. In matrix notation, we have the following identities for a closed economy CxI IO accounting framework.

$$U\mathbf{i} + e \equiv q \tag{6a}$$
$$V\mathbf{i} \equiv g \tag{6b}$$
$$V'\mathbf{i} \equiv q \tag{6c}$$

Next, behavioral relationships are indicated as follows:

$$B = U\hat{g}^{-1} \tag{7a}$$
$$U = B\hat{g} \tag{7b}$$
$$D = V\hat{q}^{-1} \tag{7c}$$
$$V = D\hat{q} \tag{7d}$$

Equation 7a defines the production requirements of commodities per industry output dollar, and equation 7b describes the industry-based technology assumption: commodities are produced by industries in fixed proportions.[1] The effect of pre-multiplication of a commodity vector or matrix by D is the transformation from commodity-space to industry-space, so $V\mathbf{i} = g = Dq$. These identities and behavioral assumptions enable these formulations:

$$q = Bg + e \tag{8a}$$
$$q = BDq + e \tag{8b}$$
$$q = (I - BD)^{-1}e \tag{8c}$$

Similarly, premultiplying Equation (8a) by D yields

$$Dq = DBg + De \tag{9a}$$
$$g = DBg + De \tag{9b}$$
$$g = (I - DB)^{-1}De \tag{9c}$$

[1]The alternative is the commodity-based technology assumption, which while not used here, could be developed in parallel fashion.

3.2 Open Economy

Equations 6 through 9 describe fully an economic system closed to trade. Analysts who use IO for impact assessment will nearly always need to reformulate the system representation to accommodate trade with the rest of the world. In the process, technical coefficients, a_{ij}, are effectively bifurcated to be equal to the sum of the domestic input per dollar output coefficient, r_{ij}, and the import coefficient, m_{ij}, or $a_{ij} = r_{ij} + m_{ij}$. This bifurcation procedure is commonly called *regionalization* when its goal is to parameterize subnational regional accounts, but the approach we use also can be implemented in similar fashion for national "regions."

Jackson's (1998) method, the first to address explicitly the regionalization of CxI as opposed to IxI national accounts, offers a number of advantages in terms of transparency in exposition, formal representation, and algebraic manipulation. It involves the modification of *Make* and *Use* tables to correspond to estimated regional output by industry, estimation of one set of regional final demand activities based on regional production levels and a second set of regional final demand activities related more directly to the economic size of the region (e.g., gross regional product relative to the national economy), and lastly, a supply-demand pooling method modified by cross-hauling estimates.[2] Jackson (1998) introduced a convenient mechanism in matrix notation for bifurcating the technical coefficients as described above, the core of which is the standardization of the *Make* table not by domestic commodity output but instead by total regional supply, which is consistent with the RSP denominator in equation 3. The effect, elaborated further below, is analagous to other "rows-only" adjustment methods, where each row is multiplied by a value between zero and 1.0 that reflects the regional supply percentage – the proportion of local commodity demand that is supplied locally, i.e., produced within the region.

Jackson's (1998) paper originally defined $\tilde{D}$ as the *Make* matrix standardized by $s = q + m$, or $\tilde{D} = V\hat{s}^{-1}$, but that formulation implicitly assumed that imports would satisfy both domestic and export demand in the same proportions. Imports that satisfy export demand without further processing, however, fall into a special category known as reexports. This is the classification for products that clear U.S. customs and are subsequently exported without additional processing or change. The U.S. B.E.A. now reports reexports separately (for 2015 - 2020 as of this writing). Beginning with the first IO-Snap

[2]The IO-Snap approach to estimating regional U, V, va, and e are laid out in Jackson (1998) and supplemented in Jackson (2020a) that modifies the original approach to foreign imports, and Jackson (2020b) that provides details on the treatment of cross-hauling.

update of 2022, which updates the state and national economic data to 2020, we report for 2015 and later the published national accounts as usual, but for national multipliers, national impacts assessments and for the regionalization procedure, we adjust the national accounts to eliminate reexports from foreign exports and imports. With this adjustment, there will be no import content in export demand, all of which will be met by regional production.

This document is an update to Jackson and Járosi (2020), where re-exports were implicitly assumed to be zero. The formulation below is largely unchanged from the earlier document, because that formulation was developed for a system where re-exports are zero. Instead of reformulating the algorithm to explicitly address reexports, our approach has been to modify the published accounts as described above, eliminating reexports from the system representation so that zero reexports is the reality and no longer an assumption.[3] We can do this because domestic industries are not involved in reexports and are unaffected by changes in quantities that are reexported. Note that the elimination of reexports from foreign exports and foreign imports results in a complete offset, such that there is no impact on the commodity output balance equation. Commodity output will be the same before and after eliminating reexports from the accounts. See the Appendix for the formal relationships.

A formulation where export demand will be satisfied solely by domestic production within the CxI framework is obtained by recasting the RPS in commodity space. Begin by specifying a a commodity-space counterpart to equation 3, as shown in equation 10. Define the units of vectors m and ex as import and export values by *commodity* (as opposed to imports and exports by industry, as in Section 3), and let

$$Q = (\widehat{\mathbf{i}V - ex + m})^{-1}(\mathbf{i}V - ex) = (\widehat{q - ex + m})^{-1}(q - ex) \tag{10}$$

where all variables share the definitions in Section 1 and refer to a set of open regional accounts. As in equation 7c,

$$D = V\hat{q}^{-1}. \tag{11}$$

Now define $\tilde{D}$ as

$$\tilde{D} \equiv D\hat{Q}. \tag{12}$$

Substituting equation 10 into equation 12, we obtain

$$\tilde{D} = D(\widehat{q - ex + m})^{-1}\widehat{(q - ex)} = D(\widehat{\mathbf{i}V - ex + m})^{-1}(\widehat{\mathbf{i}V - ex}) \tag{13}$$

[3] In IO-Snap, the national accounts that can be viewed are those that include reexports and therefore match the published versions.

Next, we draw from the standard CxI accounting relationships, beginning with the commodity balance equation.

$$U\mathbf{i} + C + G + I \equiv q - ex + m \tag{14}$$

Now multiply equation (14) by $\tilde{D}$ and substitute $D\hat{Q}$ from equation 12 on the RHS to obtain

$$\tilde{D}U\mathbf{i} + \tilde{D}(C + G + I) = D\hat{Q}(q - ex + m) \tag{15}$$

Substituting Bg for $U\mathbf{i}$, where g denotes industry output, and using equation 10, we obtain

$$\tilde{D}Bg + \tilde{D}(C + G + I) = D\widehat{(q - ex)}(\widehat{q - ex + m})^{-1}(q - ex + m) \tag{16}$$

$$\tilde{D}Bg + \tilde{D}(C + G + I) = D(q - ex) = g - D(ex) \tag{17}$$

And rearranging, we obtain

$$g - \tilde{D}Bg = \tilde{D}(C + G + I) + D(ex) \tag{18}$$

or,

$$g = (I - \tilde{D}B)^{-1}[\tilde{D}(C + G + I) + D(ex)] \tag{19}$$

The regional impact of new export demand is

$$\Delta g^{ex} = (I - \tilde{D}B)^{-1}D\Delta(ex) \tag{20}$$

the regional impact of new intra-regional final demand is

$$\Delta g^{R} = (I - \tilde{D}B)^{-1}\tilde{D}\Delta(C + G + I), \tag{21}$$

and the comprehensive impact assessment equation is

$$\Delta g = (I - \tilde{D}B)^{-1}[\tilde{D}\Delta(C + G + I) + D\Delta(ex)] \tag{22}$$

4 Summary

This document has provided an update to the foundations for a CxI version of traditional interindustry IO regionalization methods. The approach described in Jackson and Járosi (2020) assumed zero reexports, but in recognition of published reexports values, this update addresses reexports explicitly. Because reexports are independent of domestic industry production, the algorithm of 2020 remains effectively intact, but now requires the elimination of reexports from the national accounts that form the foundation for regionalization. The path from national CxI accounts to regional analytical formulations is explicit, as are supporting frameworks for impacts.

References

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Appendix A The Role of Reexports

If we were to reformulate commodity balance equation (14) to explicitly include reexports, as we see with the U.S. national accounts, we would obtain:

$$q \equiv U\mathbf{i} + C + G + I + EX - M \tag{23}$$

where EX and M include reexports, i.e.,

$$EX = ex + reexports \tag{24}$$

and

$$M = m + reexports, \tag{25}$$

we can see the expanded equation:

$$q \equiv U\mathbf{i} + C + G + I + ex + reexports - m - reexports \tag{26}$$

which, after eliminating terms leaves us with the zero-reexports identity:

$$q \equiv U\mathbf{i} + C + G + I + ex - m \tag{27}$$

Equation (27) is identical to the starting point for the zero-reexports regionalization as developed in Jackson and Járosi (2020) and reproduced in Section 3.2 , above.